Context as Assumptions
Erich Rast
erich@snafu.de

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Erich Rast
New University of Lisbon, Portugal

1 Introduction

In this paper some phenomena of linguistic context dependence are investigated from the perspective of regarding context as being constituted by the assumptions of individual discourse participants. In Section 2, a general overview of linguistic context dependence is given and a distinction between indexicals and contextuals is introduced. After this exposition some adequacy criteria, or at least reasonable rules of thumb, for modeling the linguistic context dependence of typical contextuals in a truth-conditional setting are laid out (Section 3). Finally, in Section 4 the modeling of contextuals will be addressed in a more formal setting based on simple type theory. The central idea of this section is based on the interpretation-as-abduction view introduced by Hobbs et al. (1993).

The distinction between indexicals and contextuals made in this paper has evolved from a recent philosophical debate about the nature of semantic content and the amount as to which pragmatic factors play a role in its computation. The main positions in this debate are currently semantic minimalism, see Cappelen and Lepore (2005a, 2008), Borg (2004, forthcoming) and in a special form by Bach (2006, 2007a,b,c), moderate contextualism defended by indexicalists such as Stanley (2000) and Stanley (2000, 2002), radical contextualism defended by Recanati (2004) and in another form by relevance theorists such as Sperber and Wilson (1986, 2006), occasionalism of Travis (2008), and assessment-relativism like in MacFarlane (2005b, 2007, 2008) and Lasersohn (2005). However, it is not the purpose of this article to lay out all of these positions in detail. Instead, we assume in what follows a moderate contextualist position as in Rast (2009). Many of the theses about context that will be defended below are neutral with respect to or compatible with other broadly-conceived contextualist positions, but they are more or less incompatible with occasionalism and Cappelen and Lepore’s version of minimalism. These po-
sitions will be criticized indirectly, but presenting detailed arguments against them is beyond the scope of this paper and has been done elsewhere. See for example Bach (2007a,b,c) for a critique of Cappelen and Lepore (2005a) and Borg (forthcoming) for a critique on Travis.

2 Forms of Linguistic Context Dependence

2.1 Context: A Brief Overview

Contexts are theory-dependent entities similar to propositions or electrons, and for this reason there is no such thing as the context. What a context is depends on the purpose and the intricacies of a specific theory of context. In the linguistic domain, broadly-conceived two traditions have evolved. First, based on work by Frege (1886), Reichenbach (1947), Russell (1966), and Bar-Hillel (1954) a view on linguistic contexts has become popular according to which contexts either represent these features of an utterance situation that are needed in order to determine the semantic value of indexical expressions or particular linguistic signs (tokens) of indexicals are represented explicitly. Originally having been motivated by the foundational question whether indexical context dependence is in principle reducible or not, this tradition has shifted to a more descriptive perspective and in a sense culminated in the work of Kaplan (1988), whose type-based two-dimensional semantic approach has been very influential. In these accounts based on double-index modal logics the meaning of indexicals is represented by a function from context parameters to intensions that are in turn functions from indices to extensions (see Figure 1). According to an alternative view that has been popularized by Perry in a vast number of publications, see for instance Perry (1977, 1979, 1997, 1998, 2005), the dependence of indexicals on features of the utterance situation is expressed by explicitly quantifying over reified utterances. Broadly-conceived token-based approaches like Perry’s go back to Burks (1949) and Reichenbach (1947).²

A quite different view on linguistic context can already be found in work by linguists like Jespersen (1922), Bühler (1934), and Fillmore (1972), where context is investigated from a more general linguistic and cognitive perspective. Formal theories of cognitive contexts have been developed much later based on

\[
\text{Linguistic Meaning} + \text{Context} \Rightarrow \text{Content} \\
\text{Content} + \text{Index} \Rightarrow \text{Extension}
\]

Figure 1: Two-dimensional semantics following Kaplan (1988).
ideas by Stalnaker (1978) and their subsequent implementations in dynamic semantic frameworks such as Kamp and Reyle (1993), Heim (1983), and Stokhof and Groenendijk (1991) (among others). Context is in this tradition constituted by certain doxastic or epistemic states of discourse participants and these are updated when an agent obtains new information, accepts an utterance, or silently accommodates a presupposition. While Stalnaker (1978) was primarily interested in modeling the common ground between discourse participants, i.e. the communicative assumptions that they mutually share at a given time, in a more general approach assumptions, beliefs, or knowledge of individual discourse participants may be modeled explicitly in order to to be able to faithfully represent cases of communication success and failure. In dynamic models context can also be considered in a more abstract fashion as a representation of content that is updated by context-change potential of linguistic expressions.

A third tradition of dealing with contexts has started in Computer Science with McCarthy (1993). In Artificial Intelligence research, contexts are often reified and made available within the object language, making it possible to reason explicitly about contexts within the object language and formulate so-called bridge rules for transitions between them. Work by Giunchiglia and Serafini (1994), Serafini and Bouquet (2004), Buvač et al. (1995), Buvač (1995), Buvač (1996), and Thomason (2003) exemplifies this tradition. The way context is treated in these languages is similar to the way it is treated in descendants of Kaplan's Logic of Demonstratives insofar as contexts act as reference points, but since it is possible to explicitly formulate rules between contexts by using the full power of first- or even higher-order quantification these languages are generally more expressive than mere double-index modal logics.

2.2 Linguistic Distinctions

It is fair to say that the logical toolbox available to the average philosopher or linguist has increased tremendously during the past few decades and in light of sheer number of options for dealing with context formally in a truth-conditional setting some independent criteria are needed for determining which sorts of context dependence are at play in a given linguistic example. First and foremost, linguistic context dependence has to be detected. According to simple context shifting arguments (CSAs) a sentence $\phi$ is semantically context dependent if an utterance of it is true and another utterance of it is false. Practically all sentences of any language are context dependent in this way, because almost all languages have tenses. A second question to ask is whether the expression in question semantically depends on the deictic center, i.e. the speaker, his
location, body alignment, his pointing gesture (if there is one), and the time
at which the utterance is made. These features comprise the narrow context
(Perry 1998) and an expression that semantically depends on these features is
indexical. Whether or not an expression is indexical in this sense is implicitly
known by a competent speaker and can be made explicit by the semanticist when
he is informed by competent speakers. There are also a number of tests that
can be used as a rule of thumb to detect indexicality in a sentence, although
they do not work reliably in each and every case. For example, in order to
report (1) Alice: I am hungry in indirect speech I needs to be replaced by he,
whereas (2) Bob: Alice says that I am hungry obviously doesn’t report (1). In
contrast to this, (3) Alice: John is tall can be adequately reported as (4) Alice
has said that John is tall in indirect speech without any need for additional
transformations. This shows that I is indexical and tall is not, although both
expressions are semantically context dependent.4

To fix some terminology, let a context that represents features of the deictic
center needed for the saturation of indexicals be an utterance context and
one that represents doxastic or epistemic states of discourse participants be
a doxastic context. Expressions that semantically depend on the utterance
context will from now on be called indexicals. To these belong for example I,
you, here, a special and relatively rare use of actually, all absolute tenses, and
also demonstratives such as this or overthere uttered with an accompanying
pointing gesture. Other cases of context dependence cannot be explained by
a dependence on given deictic features of the utterance situation and, as will
be laid out further below, are subject to being interpreted on the basis of the
doxastic context of an agent. These expressions will from now on be called
contextuals. Note that most indexicals are also contextuals. For example, the
boundaries of the time interval denoted by now are not determinable from the
time of utterance or any other objective feature of the utterance situation and
the same holds for the boundaries of spatial indexicals like here.5

Although many indexicals are also contextuals in the sense that a certain
relevant feature of the deictic center is needed for but does not suffice for fixing
their semantic value, indexicals, demonstratives, and anaphora form in many
respects well-distinguishable and special classes of expressions that can be sub-
categorized according to further criteria like the respective dimension (temporal,
spatial, grammatical person, modality) or the distinction between endophoric
and exophoric context.6 In contrast to this, contextuals do not form a homo-
geneous class and are merely defined ex negativo. Some of them such as tall
require a semantic ingredient when they occur in a syntactically complete sen-
tence, whereas others such as to have breakfast seem to only suggest certain default interpretations like having breakfast on the day of utterance while their use in a tensed sentence also expresses some literal meaning, for instance (5) John had breakfast expresses there is a time before the time of utterance at which John had breakfast. One may speak of primary context dependence in the first case and secondary context dependence in the latter.

3 Adequacy Requirements

In this section, a number of desiderata for the adequate modeling of linguistic context dependence will be laid out. Not all cases of linguistic context dependence will be considered, though, and for example anaphora will be excepted because their linguistic behavior has been studied in detail by semanticists in dynamic settings like DRT or DPL and their explicit dependence on the endophoric context makes them rather special in contrast to other contextuals. Likewise special and not considered in what follows are uses of indexicals in narrative contexts, i.e. when a story is told, and text-deictic expressions like former and latter.7

3.1 Utterance contexts cannot be reduced to doxastic contexts and vice versa

Utterance contexts cannot be reduced to doxastic contexts and vice versa if semantic and pragmatic adequacy is desired. It is fairly trivial to show that the first direction of this thesis holds. Suppose, for example, that Alice believes it is 2pm whereas it is in fact 1pm, and utters (6) Alice: It is now 2 o'clock. With respect to the meaning of now the utterance content is underdetermined in the sense that it does not specify explicitly by linguistic means whether 2am or 2pm is meant and the boundaries of the time interval denotes by Alice's use of now are vague and not further specified by any linguistic meaning rule. There are also interpretations of now in which the boundaries are fairly large, for example in (7) Carla earns much more now than she used to 10 years ago. However, a reasonable interpretation of (6) is constrained by general worldknowledge according to which the boundaries of the indexical in (6) are much smaller. Suppose that on the basis of their background knowledge all discourse participants agree that (6) is true in the given situation if the time of utterance was 14:00 hours ± 2 minutes.8 Then (6) is clearly false and Alice is mistaken about the denotation of her use of now. Neither her epistemic state nor her
referential intentions determine that denotation. Features of the deictic center are given independently of the epistemic states of discourse participants.

The other direction of the thesis is more complicated, as there are seemingly many ways to 'objectify' aspects of doxastic context. First, one might attempt to simply store a relevant aspect in context parameters of a double-index modal logic. From a purely logical point of view, almost anything can be stored in a parameter according to which truth is relativized and for some technical purposes enriching parameters might make sense. However, the way in which relevant features of epistemic states are encoded formally should properly reflect the role they play in the resolution of context dependence. Beliefs and assumptions of agents don't generally determine missing ingredients of contextuals, because the (deep) interpretation of contextuals is sometimes optional, and moreover beliefs and assumptions are generally individual. For example, particularly when uttered with verum-focus, a speaker might intend (5') John had breakfast to be interpreted according to its literal meaning rather than its usual default interpretation. In this case nothing is missing that could be stored in a context parameter.\(^9\) It is also crucial to notice that referential intentions of speakers are not part of the context and generally are not adequate for determining the truth-conditional contribution of indexical contextuals.\(^10\) If for example Bob points to the K2 while intending to refer to the Mount Everest (8) Bob: This is the highest mountain on earth is false, just like in example (6), since the pointing gesture picks out the K2 instead of the Mount Everest.

There doesn't seem to be any principal counter-argument against yet another variant of parameter-based contextualism, according to which the contextual variation of a contextual is expressed by using different, suitably enriched parameters of a double-index modal logic. For example, relativists like Lasersohn (2005) have suggested to put a judge into the index parameter, thereby allowing for two people to disagree about the same semantic content of an utterance containing a predicate of personal taste without one of them being at fault.\(^11\) The general usefulness of these kind of theories is questionable, though. Contextual variation is in these theories merely expressed formally without explaining how an agent arrives at a particular interpretation, and when the interpretation of contextuals is modeled by resorting to parameters, context or index parameters are multiplied respectively: one parameter is needed for the deictic center and other parameters for representing different interpretations and what the speaker has in mind. As a result, the connection between communicative assumptions and beliefs of discourse participants and their preferred interpretation of an utterance at a given time is left unexplained. That being said, there
is nothing wrong with having different parameters for different interpretations, assessments, or evaluations of an utterance, as long as one is only interested in expressing or encoding contextual variabilities in a logical language.

3.2 Knowledge is not indexical

Both contextualism and relativism about knowledge or knowledge ascriptions have been defended recently.\textsuperscript{12} While a general critique of these positions is beyond the scope of this paper, there is a strong argument against a crude form of indexicalism of strong knowledge. Let there be a weak epistemic context $c_w$ and a strong one $c_s$, let $Kp$ stand for 'it is knowable that $p$', and $M, c \vDash \phi$ express the fact $\phi$ is true with respect to context $c$ in a model $M$. Now assume that $p$ is itself not sensitive to epistemic contexts. Given all that, according to the indexicalist premise it can be the case that (i) $M, c_w \vDash Kp$ and (ii) $M, c_s \vDash \neg Kp$. But from (i) it follows by factivity of knowledge that $M, c_w \vDash p$. Since $p$ is by assumption not sensitive to epistemic contexts, it thus also the case that $M, c_s \vDash p$. Given all that, the last and crucial step of the argument is as follows: The fact that $p$ holds in the strong context and the fact that this fact in turn can be derived on the basis of uncontroversial logical principles, the factivity of knowledge, and the indexicalist premise taken together should more than suffice as a justification for the claim that it is also knowable in $c_s$ that $p$, i.e. for establishing $M, c_s \vDash Kp$, in any particular case. This contradicts with the contextualist assumption (ii).

Some epistemologists apparently don't like this argument. They tend to attack it either by resorting to an alternative notion of contexts or by attacking the last inference step. Regarding the first counter-argument, notice that the original argument is independent of the actual formal modeling of the contexts in question and so it does, for instance, not help to consider contexts as sets of possible worlds instead of simple reference points.\textsuperscript{13} The argument does not rest upon any assumptions about the structure of contexts at all. The argument applies to any sort of determinative context, i.e. to any sort of context that partly determines the truth or falsity of a knowledge attribution such that (i) and (ii) may hold at the same time and within the same model. Second, it is hard to see how the very fact that some claim can be derived by logical principles from acceptable assumptions cannot be a valid justification. Conversely, the justificational value of such a fact should be stronger than any empirical claim. It is easy for an agent to ascertain in any particular case that the embedded proposition is true in the strong context when it is already known in a weak context. Hence, the agent certainly has good reasons to believe
that it holds in the strong context and since the embedded proposition is true
and the justification is correct, according to the justified true belief view the
agent also knows that the proposition holds. The only thing that would keep
an agent from knowing the embedded proposition would be a lack of awareness
about the logical principles that govern strong knowledge or a lack of inferential
skills in general. After all, a heavily resource-bound agent might not even
be able to recognize simple instances of modus ponens as correct inferences.
However, it is not easy to see how switching to resource-bound agents could
salvage epistemic contextualism, because the resulting kind of contextualism
would be fairly trivial. In this view, the agent would simply fail to recognize
that it follows from the fact that he knew the embedded proposition in the weak
context that the embedded proposition is also true in the strong one. In that
case it is still knowable in the strong context that the embedded proposition
holds and we should be able to convince the agent of this fact as easily (or hard)
as it might be to convince someone of the fact that modus ponens is a valid
inference scheme.

What lesson should be drawn from this argument? One might be tempted
to consider the verb to know a contextual as laid out above. If to know in-
deed worked exactly in parallel to expressions like tall, then stronger or weaker
readings of it would be obtained by interpreting the respective knowledge as-
cription, and a statement of the form A knows that p would be semantically
underdetermined in a sense that will be laid out in more detail in the next
sections. No such readings seem to be available, though, and so invariantism is
a better response. Strong knowledge might have its place in epistemology only
as an ideal limit to which justified beliefs converge ideally.

3.3 Deep interpretation is sometimes optional and sometimes mandatory

Bach (2004, 2005) has argued that the recipient does not always need to find
a missing ingredient of a contextual. As mentioned earlier, in (5) John had
breakfast a default interpretation is indicated according to which John had
breakfast on the day of utterance, but the literal meaning of the sentence can
be prevalent in a given conversational situation. For example, when previously
someone has mentioned that John has never had breakfast in his life, Alice may
reply with (5) and add that she has seen John having breakfast last week, al-
though it was a quite hasty one. Another example is (9) Alice bought a car.
From the point of view of lexical semantics buying something involves a legally
binding transfer of a property between a buyer and a seller at a certain price,
since otherwise the act of buying cannot be distinguished from similar acts like borrowing or stealing. But many times when (9) is uttered, the recipient does not need to determine a specific seller or price in order to understand what (9) says or what the speaker intended to say by uttering (9). Finding a specific contextual ingredient is optional in such a case, but by virtue of semantic competence a recipient must still know implicitly that buying something involves a purchased object, a buyer, a seller, and a price. When a specific ingredient is determined by the recipient, this is from now on called deep interpretation. In contrast to this, the existential completion that for the above example may be paraphrased as There is a seller and there is a price at which Alice bought a car at some time in the past is the result of partial interpretation. If Bach (2007d) is right, partial interpretation by existentially quantifying over open argument places is optional as well, because sometimes other than existential quantifiers might yield the desired interpretation. It is, however, unclear as to the time of this writing under what circumstances contextual sentences can be interpreted using another than the existential quantifier. For example, it seems that (10) John ate cannot be interpreted as (11) John ate most of the cookies and (10) cannot be uttered felicitously to convey this interpretation.

Sometimes deep interpretation also seems to be mandatory. For example, assuming some place of arrival when (12) Alice arrived last week seems to be required by the conventional meaning of to arrive.¹⁴ In other words, there is a sense in which someone who interprets (12) as Alice has arrived at some place during the week before the utterance of (12) has not fully understood (12) in the given conversational situation, although he has grasped its linguistic meaning, whereas the same cannot be said about the existential completion of (9). The fact that to arrive has an indexical and a non-indexical reading similar to left and right might account for this difference. While certain contextuals are not indexical in the narrow sense of semantically depending on the deictic center, they still semantically depend on features of another center in the same way as indexicals.

3.4 Doxastic contexts are constituted by assumptions

Doxastic contexts are in a sense given by the belief states of discourse participants, but as plenty of research on presuppositions has shown not directly. Stalnaker (1978, 2002) and many others have argued that in order to account for the silent accommodation of presuppositions doxastic contexts are comprised of mutual assumptions of discourse participants, i.e. their common ground. Consider the following example due to von Fintel: (13) I am sorry that I am
late. I had to take my daughter to the doctor. Among the presuppositions of these sentences is the existential presupposition that the speaker of (13) has a daughter. It is fairly obvious and a common phenomenon that a hearer doesn't need to know that the speaker has a daughter in order to fully understand (13), because he can simply add this presupposition to his belief base on the fly, thereby maintaining the common ground.

However, mutual assumptions alone do in a trivial sense not suffice for modeling discourse in general, if the model is supposed to reflect not only what happens during successful, but also what happens during unsuccessful communication. What happens if the hearer doesn't accommodate the presupposition? Clearly, the assumptions of discourse participants have to be modeled on an individual basis as well, and from these epistemic states the common ground can be computed at any time. Moreover, although mutuality plays a crucial role in explaining certain cases of Gricean, interdependent reasoning processes by means of which an agent arrives at an interpretation, its role for everyday communication has been exaggerated in the past. Often a hearer just maintains a model of what the speaker appears to believe and on the basis of this model interprets his utterances and accommodates presuppositions accordingly. It should also be remarked that assumptions, as opposed to beliefs, play a less important role for the interpretation of contextually than for dealing with presuppositions. The missing ingredient of a contextual is marked as such in the lexicon but under usual circumstances no particular instance of this missing ingredient is presupposed or implicated. For example, when someone interprets (3) John is tall he cannot accommodate the missing comparison class, because it is not indicated by the utterance at all. An agent arrives at a comparison class by taking into account the question under discussion (QUD). Is the utterance about playing basketball and John plays basketball? Then the members of his team might be a preferred comparison class.

In a simplified view assumptions without higher mutuality constraints can be generated from what the recipient believes about what the message sender believes. Ideally, these beliefs should be compartmentalized in dependence of the QUD. In practise, it is an open question how agents compartmentalize beliefs on the basis of what has been said so far and how this dependence may be modeled in a logical setting under ideal rationality assumptions.
4 Notes on Representing Contextuals

The remainder of this paper addresses the question how to represent contextuals in a formal, truth-conditional setting. Since any semantic representation of linguistic expressions depends to a large extent on the framework used, the following considerations primarily serve as methodological suggestions and are intended to further explicate the points made in the last section.

4.1 Using Free Variables

We take a closer look at the semantic content of some contextuals in a simple type theory called $T\bar{T}$ (see Appendix), whose only difference to standard type theory is that a special notation is used to give functions a second extension. In case of a function $A$ from entities of some type to truth-values $\{1, 0\}$, $\neg A$ is interpreted as inner negation. This means, for example, that for an ordinary, non-intensional predicate $P_{(st)}$, $\neg P(a) \land \neg \neg P(a)$ may be true in a model, thereby representing the fact that $P$ is not applicable to $a$. Consequently, semantic objects of type $s$ can be regarded as situations as opposed to worlds, because from the fact that $\neg P_{(st)}(a)$ it does not follow that $\neg P_{(st)}(a)$, whereas the opposite direction holds, and the inner negation must be considered the ‘genuine’ negation. In general, this makes the logic very similar to a partial logic that corresponds to a 3-valued Kleene system but without giving up bivalence or having to introduce additional functors.\footnote{Two-dimensional semantics can be implemented in this framework by combining terms of type $(s(st'))$ for various types $T$, finally yielding some term of type $\tau$. The type $(s(st'))$ for sentences is abbreviated $\tau$ and the type $(s(se))$ for intensional objects is abbreviated $\epsilon$. In what follows, the variable $u$ is used for the utterance situation and $s$ for what may be called the topic situation, i.e. it stands for the situation that is implicitly described by the utterance.}

To give an example, let (14) $\lambda u s.\text{speaker}(u)$ be an expression of type $\epsilon$ for the English first-person pronoun, (15) $\lambda s.\text{PRES}(u, s) \land P(u)(s)$ for the present tense, where $\text{PRES}(s_1, s_2)$ is true if the start times of $s_1$ and $s_2$ are equal and $s_2$ ends when $s_1$ ends or later, and (16) $\lambda x_2 u.\lambda s.\text{wait}(s, x(s, u))$ a lexicon entry for the verb to wait. The sentence I wait is then analyzed as (17) $\lambda P_s.\lambda u s.\text{PRES}(u, s) \land P(u, s)[\lambda u s.\text{speaker}(u)\lambda x_2 u.\lambda s.\text{wait}(s, x(s, u))]$, which reduces to (18) $\lambda u s.\text{PRES}(u, s) \land \text{wait}(s, \text{speaker}(u))$.\footnote{If what has been said so far is correct, the context dependence of tall in (3) John is tall cannot be adequately expressed in the same manner as a function of the utterance situation like in (19) $\lambda u s.\text{PRES}(s, u) \land \text{Tall}(s, j, f(u))$, where...}
\textit{f} is a function from a situation-type variable to a comparison class (viz. corresponding predicate), since the missing comparison class is not provided by a shared context. It is better to represent the missing comparison class as a free variable instead, as in (20) $\lambda u \lambda s. PRES(s, u) \land \text{Tall}(s, j, C)$. For the present purpose of investigating interpretations of utterances, a free variable must at some point be bound by an existential quantifier in contrast to the usual practice in mathematical logic of assuming implicit universal quantification. Formula (22) $\lambda u \lambda s. \exists C[\text{PRES}(s, u) \land \text{Tall}(s, j, C)]$ represents the \textit{existential completion} of (20). Existential completion plays a crucial role in keeping interpretations conventional from a logical perspective, because it allows one to avoid explicit representations of incomplete content such as structured propositions with all of the problems that are associated with such approaches.\(^{18}\)

Admittedly, not all missing ingredients of contextuals have to be represented as a free variable. First, it would, of course, also be possible to bind the variable by a $\lambda$-operator and delaying the evaluation until the end of semantic composition. This would significantly complicate syntactic and semantic construction, though. Secondly, it will be shown in the next section that dependences on the utterance and topic situation respectively can be modeled to some extent by quantifying over each of these situations like a $\Box$-modality in modal logic. When quantifiers are properly relativized to these situations, for example by a domain predicate of type $(\epsilon(st))$, even quantifier domain restriction, nominal restrictions, and other implicit domain dependences like that of spatiotemporal indexicals can be dealt with. However, when possible open variables ought to be preferred over implicit dependences on the underlying semantic objects, because they allow for a more controlled modeling of deep interpretation.

### 4.2 Belief and Assumptions

A modal notion of rational belief can be expressed in $\mathcal{T}\mathcal{Y}$ by explicitly representing an accessibility relation of the appropriate kind. Let \textit{Believe} stand for (23) $\lambda x \lambda y \lambda P \lambda u \lambda s. A(s, x(u, s)) \land \forall s' R(s, s', x(u, s)) \rightarrow P(u, s')$, where it is stipulated that in each situation the agent predicate $A$ is only true for finitely many agents in $D_a$. As long as it is only used for a finite number of agents, $R$ can be regarded a collection of finitely many binary accessibility relations over situations like in a normal modal logic. Familiar $KD45$ belief can be obtained
Figure 2: Generating the assumption \( \neg P \) by revising the accessibility relation for agents \( \alpha \) and \( \beta \).

by adding the usual constraints:

\[ (D.) \forall x \forall s_1 \exists s_2 R(s_1, s_2, x) \quad \text{Seriality} \]

\[ (4.) \forall x \forall s_1 \forall s_2 \forall s_3 [(R(s_1, s_2, x) \land R(s_2, s_3, x)) \rightarrow R(s_1, s_3, x)] \quad \text{Transitivity} \]

\[ (5.) \forall x \forall s_1 \forall s_2 \forall s_3 [(R(s_1, s_2, x) \land R(s_1, s_3, x)) \rightarrow R(s_2, s_3, x)] \quad \text{Euclideanness} \]

There are basically two routes to go for the modeling of assumptions. First, one may use an independent modality for them. Since on one hand from the fact that a hearer assumes that the speaker has \( \phi \) in mind when making an utterance it prima facie doesn't seem to follow that the hearer also assumes that he assumes that the speaker has \( \phi \) in mind (and likewise for the negative case), (4.) and (5.) for positive and negative introspection could be dropped for this modality. On the other hand, these rationality assumptions don't seem to do any harm. We will go the second route instead and generate communicative assumptions from a discourse participant's beliefs, because this casts more light on the connection between belief and communicative assumptions. There is also the opposite direction that leads back from an interpretation under some communicative assumptions to the formation of new beliefs and revision of old beliefs. This interesting process, which seems to involve a combination of non-prioritized belief revision and a plausibility ranking, will not be addressed in what follows.

The idea mentioned in the previous section was that a hearer generates his communicative assumptions from what he believes that the speaker believes. Now suppose \( R_2^y \) is the accessibility relation that results from revising \( R \) in such a way that all first-order beliefs of \( x \) are revised by \( x \)'s beliefs about what \( y \) believes. Implementing a revision from \( R \) to \( R_2^y \) is not trivial; the so-called paradoxes of introspective belief change have to be avoided and it has to be shown that the operation fulfills the requirements of AGM belief revision Alchourrón et al. (1985). These details go beyond the scope of this paper and at this time the desired operation shall only be characterized by means of the
following postulate that \( R \) and \( R^2 \) must satisfy:

\[
(R. \quad \forall x, y, z \forall s_1, s_2, s_3, s_4 \exists z' \forall P_{st} \left[ \left( R(s_1, s_2, x) \land R(s_2, s_3, y) \land R(s_2, s_4, z) \right) \land \left( \begin{array}{c}
R^2(s_1, s_2, x) \land R^2(s_2, s_3, y) \land R^2(s_2, s_4, z) \\
\neg R^2(s_1, s_3, x) \\
\land (P(s_2) \leftrightarrow P(s_3))
\end{array} \right] \right)
\]

Notice that the formula also quantifies over expressions of the form \( \neg P \), because they are of type \( st \). The rather simple idea behind this constraint is illustrated in Figure 2. When \( x \) believes that \( p \) but believes that \( \neg p \), then he should believe that \( \neg p \) according to the revised accessibility relation and correspondingly for the case when the first-order belief is \( \neg p \) and the second-order belief is \( p \). To assure this, it is stipulated that the link from \( s_1 \) to \( s_2 \) is cut and replaced by a link to a situation \( s'_2 \) at which the proposition has the desired truth value. This is, of course, a rather trivial revision: the agent simply makes all of his beliefs about what he believes that the speaker believes his own beliefs for the time being, including cases when he believes that the speaker is in doubt about something. Using the revised accessibility relation, \( z' \)'s communicative assumptions in \( u, s \) may be captured using (24)

\[
ASSUME := \lambda x, \lambda y, \lambda z \forall t \forall s' \exists u' \left[ \left( R_{z(u,s)}^{(u,s)}(u, u') \land R_{z(u,s)}^{(u, s)}(s, s') \right) \land (P(u', s')) \right]
\]

This formula represents a form of partial, interdependent interpretation for which there is no direct counterpart in natural language. When for example Anne says (25) \( I \) love you, (26) \( ASSUME[Bob, Anne, \lambda u, s, \exists x (Pres(u, s) \land address(u, speaker(u), x)) \land love(s, speaker(u), x)] \) can be used to represent Bob’s beliefs about what Anne believes about the utterance, including his assumptions about which indexicals in question refer to in her opinion. These assumptions can be quite relevant for Bob’s behavior, and not just his linguistic behavior. Think about a ménage à trois: Did she mean me? Or did she just talk to this idiot John? She didn’t mean both of us, did she? I’m gonna kill him . . . To give another example, (27) \( ASSUME[Bob, Alice, (22)] \) represents a limited interpretation of (3) by Bob, but as has been argued in the previous section sometimes an additional, deep interpretation seems to be mandatory.

As is well-known from the literature on belief revision, there are in principle many ways to revise beliefs. When the hearer does not actually believe what he believes that the speaker believes, he might adopt is background knowledge in many different ways in order to generate what he considers to be the speaker’s background knowledge that is required for further interpreting the utterance. Consequently, even the result of weak interpretation is not fully determined by the belief state of a discourse participant but the result of weak interpretation also depends on which actual revision operation is assumed.
Figure 3: Relation between an interpretation and an existential completion, where $C_0$ is a constant.

4.3 Preferred Interpretations

An agent’s actual interpretation implies his assumption of the existential completion (see Figure 3). Since from the lexical meaning of an utterance alone only its existential completion can be derived, deep interpretation can be described as an abductive inference step from the existential completion, via the generation of assumptions, to a narrower and more specific interpretation. The idea of regarding interpretation as abduction has been introduced by Hobbs et al. (1993), see also Stone and Thomason (2002), and implemented syntactically by assigning numerical preference values to formulas of a first-order language and their parts. To obtain a similar effect in the present setting, we define a global ordering on functions from situations to functions from situations to truth-values. Let $\leq_\pi$ stand for a collection of relations of type $(\tau(\pi))$, for each agent $\pi$, such that the usual conditions for a total order are fulfilled:

(P1.) $\forall x \forall \beta \forall \gamma \forall \delta \forall e \forall \tau \forall P \forall Q \forall R\left[ (P \leq_\pi Q \land Q \leq_\pi P) \rightarrow P = Q \right]$ Antisymmetry

(P2.) $\forall x \forall \beta \forall \gamma \forall \delta \forall e \forall \tau \forall P \forall Q \forall R\left[ (P \leq_\pi Q \land Q \leq_\pi R) \rightarrow P \leq_\pi R \right]$ Transitivity

(P3.) $\forall x \forall \beta \forall \gamma \forall \delta \forall e \forall \tau \forall P \forall Q\left[ P \leq_\pi Q \lor Q \leq_\pi P \right]$ Totality

An agent’s interpretation of an utterance of type $\tau$ with respect to the respective speaker is then the most preferred semantic content (of type $\tau$, in the present setting) that implies the utterance’s existential completion. To put this in more precise terms, in the following formula $P$ is the most preferred interpretation for agent $\pi$ with respect to another agent $\gamma$ and the existential completion $Q$: (28) $\lambda x_0 \forall y_0 \forall x_1 \forall Q_0 \forall P_0 \forall u_0 \forall z_0 \forall s_0 \forall s' \forall Z_0 \forall \tau_0 \forall \alpha_0 \forall \beta_0 \forall \gamma_0 \forall P_0 \forall Q_0\left[ (P_0 \leq_\pi Q_0 \land Q_0 \leq_\pi P_0) \rightarrow \left[ Z_0(u_0, s_0) \rightarrow Q_0(u_0, s_0) \right] \land \left[ Z_0(u_0, s'0) \rightarrow \left[ Z_0(u_0, s'0) \rightarrow Q_0(u_0, s'0) \right] \land \left[ Z_0(u_0, s_0) \rightarrow Q_0(u_0, s_0) \right] \right] \right]$. When a iota operator is added and the language is modified accordingly to deal with non-denoting terms, the
preferred interpretation $P$ may also be obtained directly as the assumed object of type $(s(st))$ that is minimal with respect to the ordering relation.

It should be clear from the discussion of contextuality in the previous section that certain cases of secondary context dependence require some additional machinery for representing common sense knowledge and an agent may at times arrive at a preferred interpretation by means of quite complex inference chains. Consider again the example of having breakfast + tense. One usually has breakfast in the morning, after having woken up. Moreover, when someone talks about a past event in the afternoon that describes a daily activity that usually takes place in the morning, it is likely that this past event took place in the morning of the day of utterance unless there is additional information that suggests another past reference time frame. (Such additional information may for example be introduced explicitly as the origin of a narrated sequence of events.) The defeasible rules needed to represent the kind of common sense knowledge that leads to the conclusion that an utterance like (5) John had breakfast is to be understood as (9) John didn't have breakfast in the morning of the day of utterance of (5) may be implemented by using the sphere model of Grove (1988), which is also based on a total ordering of semantic entities. But it should be clear that an extensive amount of general, defeasible background knowledge is needed in order to make the complete inference chain explicit. Likewise, in order to arrive at preferred interpretations of (6) and (7) one has to resort to the QUD and a lot of common sense assumptions about the usual precision of talking about the time of the day, the periods during which salaries remain constant, how average salaries are usually measured, and so on. A fully adequate modeling of genuine contextuals can become a rather complex business and potentially involves a lot more machinery known from AI research.

Perhaps considering some limited form of interpretation on the basis of modalities, such as the above ASSUME function, is thus a reasonable middle-ground between a misleading and at times inadequate two-dimensional semantics for contextuals and going into the details of deep interpretation. By making spatial and temporal expressions sensitive to situations, i.e. enriching situations with temporal and spatial structure, this kind of interpretation operation can at least adequately encode—although not explain—the dependence of contextuals like spatiotemporal indexicals (including tenses) or expressions as in (9) and (12) on the doxastic context.
5 Conclusion

The main goals of this paper were on one hand to show that indexicality needs to be carefully distinguished from other forms of linguistic context dependence that are based on semantic incompleteness and on the other hand lay out some adequacy criteria for dealing with contextuals in a truth-conditional setting. By looking at ways to model the existential completion of an utterance and to compute an agent’s communicative assumptions at a given time from his beliefs different forms of ideally rational interpretation can be described. A crucial problem for modeling deep interpretation is, however, how to explain the preference relation that yields an agent’s preferred interpretation. In an ideally rational approach this relation has to be connected to the broader issue of how an agent deals with evidence obtained from sources of varying reliability. No general theory of how to connect preferred interpretation with this sort of information is available yet, and it is likely that this connection can only be explained and described in a satisfactory way by taking into account related research on fine-grained and compartmentalized belief, the QUD, and default reasoning.
Appendix: Language $T\tilde{y}$

Syntax

Types. Base types are $e$ for entities in $D_e$, $s$ for situations in $D_s$, and $t$ for truth-values in $\{T, F\}$. If $\alpha, \beta$ are types, then $(\alpha\beta)$ is a type. Nothing else is a type. $D_t = \{1, 0\}$.

Terms. We assume a fixed vocabulary of expressions, using $x, y, z$ for variables of type $D_e$ and $s, u$ and indexed variants for variables of type $s$. An expression of base type $\alpha$ is a term of type $\alpha$. If $A$ is of type $(\beta\alpha)$ and $B$ is of type $\beta$, then $(AB)$ and $(BA)$ are of type $\alpha$. If $x$ is a variable of type $\beta$ and $A$ is an expression of type $\alpha$, then $(\lambda x.A)$ is a term of type $(\beta\alpha)$. For each pair of terms $A, B$ of type $\alpha$, $(A = B)$ is a term of type $t$. Familiar infix notation may be used for the standard logical connectives $\lor, \land, \leftrightarrow, \rightarrow$. The binder notation will be used for standard quantifiers, i.e. $\forall x A$ is written instead of $(\forall(x.A))$. Traditional operator syntax will also be used in many places, types and parentheses are sometimes omitted, and implicit $\beta$-conversions are allowed for better readability. This means that for instance $\text{Hungry}(s, Alice)$ may be written instead of $((((\lambda s, (\lambda x_e.\text{Hungry}(s, s(e))))s_x)e_x)s_x) Alice_x$. A term of the form $\neg A$ is the inner negation form of $A$ and there is no inner negation form of an inner negation form.

Semantics. A non-standard $T\tilde{y}$ frame consists of a set containing sets $D_\alpha$ for each base type $\alpha$ and some sets $D_{(\alpha\beta)} \subseteq D_\beta^\alpha$ for each compound type $(\alpha\beta)$. We write $g$ for an assignment and $g[x/a]$ for the assignment that is the same as $g$ except that $g(x) = a$. A generalized Henkin model $M$ for $T\tilde{y}$ is a tuple $\langle F, [] \rangle$ consisting of a non-standard frame $F$ and an interpretation function $[]$ that in dependence of a variable assignment $g$ maps terms to their denotation according to their type as follows:

1. $[[x_\alpha]]^M_\beta = g(x)$ if $x$ is a variable, where $g(x) \in D_\alpha$
2. $\left[[\left(A_i \land (\gamma(y)) B_i\right)]\right]^M_\beta = 1$ if $[A]^M_\beta = 1$ and $[B]^M_\beta = 1$; 0 otherwise.
3. $\left[[\neg A]\right]^M_\beta = 1$ if $[A]^M_\beta = 0$; 0 otherwise.
4. $\left[[\left(A_{(\beta\alpha)\beta}\right)]\right]^M_\beta = [A]^M_\beta ([B]^M_\beta)$, where $[A]^M_\beta \in D_{(\beta\alpha)}$; likewise for terms of the form $(B_\beta A_{(\beta\alpha)})$.
5. $[[\lambda x_{\beta} A_\alpha]]^M_\beta$ is that function $F$ in $D_{(\beta\alpha)}$ such that for any $a$ in $D_\beta$, $F(a) = [A]^M_\beta[x/a]$. 

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6. \([[(\forall x. A)]^M,\beta] = \bigcap_{\alpha \in D_\alpha} [A]^{M,\beta}[\sigma/\alpha].\)

**Inner Negation Constraint.** \([A(\alpha,\beta)]^M,\beta \cap [\sim A(\alpha,\beta)]^M,\beta = \emptyset\) for any expression \(A\), i.e. a positive term \(A\) and its inner negation form \(\sim A\) have distinct extensions.
Notes

1See Stojanovic (2008) and Borg (2007, forthcoming) for overviews.

2Token-reflexive analyses can also be found in earlier work by Peirce and Russell but not with the same amount of systematicey as that of Reichenbach (1947). Although Perry speaks about token-reflexive meaning, his account is strictly speaking utterance- and not token-based (see Perry (2003)).

3Cappelen and Lepore (2005a) have terminologically introduced CSAs merely to criticize them, but we agree Bach (2007b,a) that their arguments have remained inconclusive. Notice that according to Comrie (1985) there are some languages in which tenses are not grammatically realized (e.g. Burmese) or in which not all of them need be grammatically realized (e.g. Mandarin Chinese). Nevertheless, suitable temporal relations between the reported event or situation and the time of utterance are still required from a semantical point of view.

4The test was devised by Cappelen and Lepore (2005a) for checking whether an expression is context dependent in general, but it obviously only separates expressions that semantically depend on the deictic center from others. Contrary to what Cappelen and Lepore (2005b) has claimed, it is the semanticists’ job to determine whether or not tall is relational. Just like and cannot be regarded as a unary junctor (not even in a fully curried language like $\mathcal{T}$ in the Appendix) no sensible non-relational account of tallness can be given.


6See Rast (2007, Ch. 5).

7In contrast to ordinary contextuals like tall or enough, anaphora and genuine text-deictics seem depend to large extent on the grammatical, rhetorical, and informational structure of the previous discourse in addition to how it has been interpreted so far.

8For the sake of the current argument, the potential ‘higher-order’ vagueness of the ± margins or cases when discourse participants assume different standards of precision can be ignored. It is assumed in the above example that all discourse participants agree on the margins and that they are much smaller than one hour. From a more philosophical angle one could also claim that expressions like now or 2pm denote instants in time rather than time intervals and the above interpretations are only adequate when Alice is considered as speaking loosely. As interesting as it may be from a philosophical perspective about time, this view is not helpful when doing natural language semantics. Clearly people do not have such strict standards in ordinary conversations.

10 See (ibid.), Bach (2009).

11 Note that relativism like that of MacFarlane (2007, 2008) is quite a different story; here, a metaphysical claim about the truth or falsity of utterance content at different evaluation times is made and whether this view is adequate hinges on metaphysical arguments.


13 I’m thankful to Manuel Rebuschi for having made this suggestion and a fruitful discussion.

14 Many thanks to Richmond Thomason for having brought this to my attention.

15 This view goes back to non-traditional predication theory of Siewojew (1970), Siewojew and Wessel (1975), Wessel (1989). Some philosophers and logicians don’t like it, because it cannot be readily extended to deal with to quantified statements and moreover one or both of ⊥ and ¬ might no longer satisfy ones favorite criteria for negation. Non-traditional predication theory is nevertheless useful in order to make the talk about situations adequate without going fully partial. See Muskens (1995) for a genuine partial type theory.

16 Details of the tense logic and underlying interval relations cannot be addressed here and the reader is referred to Allen (1983), Ladkin (1987) and van Bentham (1991) for more information.

17 In contrast to this, the present tense predicate \( PRE S \) is indexical and therefore does depend on \( u \). A crude definition for tall could be given as (21) \( Tall := \lambda u \lambda s \lambda x \lambda C. \text{most } y(C(s, y) \land \text{height}(s, y) < \text{height}(s, x)) \), where the quantifier and function names are self-explanatory. These details don’t matter in what follows.

18 Apart from logical problems related to attitudes towards genuine incomplete content—after all, one needs to specify the logical consequences that follow from ones having such an attitude—structured propositions also tend to lure philosophers of language into metaphysically dangerous parlance, talking as if there was an ethereal "third realm" of meanings.

19 A more general constraint would quantify over objects of type \( (sT) \) for any type \( T \) and replace \( P(s) \leftrightarrow P(s) \) with \( P(s') = P(s) \).
References


Bach, K.: 2007a, From the strange to the bizarre: Another reply to cappelen and lepore, retrieved in may 2008 from http://userwww.sfsu.edu/~kbach/, University of San Francisco, Department of Philosophy.


Borg, E.: 2007, Minimalism versus contextualism in semantics, in G. Preyer and
G. Peter (eds), Context Sensitivity and Semantic Minimalism: Essays

Borg, E.: forthcoming, Semantic minimalism, in L. Cummins (ed.), The Prag-
matics Encyclopedia, Routledge.


Burks, A.: 1949, Icon, Index and Symbol, Philosophical and Phenomenologi-
cal Research 9(4), 673–689.


Buvač, S., Buvač, V. and Mason, I. A.: 1995, Metamathematics of contexts,
Fundamenta Informaticae 23(3).

Buvač, S.: 1996, Quantification logic of context, Proceedings of the Thir-
teenth National Conference on Artificial Intelligence.


Cappelen, H. and Lepore, E.: 2005b, A tall tale: In defence of semantic minimal-
ism and speech act pluralism, in G. Preyer and G. Peter (eds), Contextualism
in Philosophy: Knowledge, Meaning, and Truth, Oxford University Press.

Cappelen, H. and Lepore, E.: 2006, Shared Content, in E. Lepore and B. Smith
(eds), Oxford Handbook of Philosophy of Language, Oxford University

Cohen, S.: 1990, Skepticism and everyday knowledge attributions, in M. D.


DeRose, K.: 1996, Relevant alternatives and the content of knowledge attribu-

DeRose, K.: 2009, The Case for Contextualism: Knowledge, Skepticism and

mantik und generative Grammatik I, Athenäum, Frankfurt a.M., pp. 147–
174.


URL: http://www.phon.ucl.ac.uk/home/PUB/KPL/02papers/wilson_sperber.pdf


